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Prediction of the droplet size and velocity joint distribution for sprays

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Abstract

This work addresses the development of a mathematical model to predict the joint distribution for both size and velocity of the droplets in sprays, based on the maximum entropy formalism. Using this joint distribution, models to obtain separated distributions for size and velocity of sprays are also presented. Correlations for the average velocity for both pressure jet and airblast atomisers, based on assumed profiles in the atomiser gun, are obtained as a function of easily measurable parameters. Several distributions for different types of atomisers are then predicted. Agreement between available data for the velocity distribution and the corresponding predictions is satisfactory. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Sprays; Droplets velocity/size; Mathematical modelling; Joint distribution

1. Introduction

Atomisation of liquids constitutes a technology presently used in almost all industrial operations. It covers a broad range of applications such as evaporative cooling, combustion systems, air/gas conditioning, fire suppression, agriculture and spray drying. Due to this wide range of applicability, interest in the size and velocity distributions of droplets in nozzle sprays has increased during the last two decades. For energy generating systems, for example, the interest is partly due to the combined effects of the relatively low price of residual fuel oils and the growing concern about pollutants emissions, the latter being influenced by the fineness attained during the atomisation process. In fact, Yuan et al. [1] presented some results concerning the effect of the spray fineness on the particulate emissions in a confined oil-fired combustor. In that work, the authors concluded that the particulate concentration at the combustion chamber could be reduced by about 60% by increasing the spray quality through the reduction of the Sauter mean diameter of the atomised spray by about 50%.

In most common practical applications, atomisation is achieved by either exposing a slow moving liquid to a high velocity gas stream, as in airblast atomisers, or conversely, by exposing a fast moving liquid to a slow moving gas, as in pressure jet atomisers. For the latter, a high velocity stream of liquid is injected into a stagnant or low velocity atmosphere inducing atomisation by the combined effects from aerodynamic forces, caused by the relative velocity of the two streams, together with those from hydrodynamic forces, originated by turbulence and disruptive forces within the liquid itself. In turn, airblast atomisers currently found in actual spray systems can be divided into two different types: plain jet atomisers and pre-filming atomisers. In the first type, the liquid is injected as discrete round jets into a high velocity coaxial gas flow, while in pre-filming atomisation, the liquid is spread into a thin sheet prior to the contact with the gaseous stream. The above-mentioned processes are comprehensively described in the works of Chigier [2] and Williams [3].

Both pressure and airblast atomisation processes may lead to equally fine sprays. However, the velocity distribution at the burner exit of the atomised liquid is strongly dependent on the atomisation process.

As mentioned earlier, a very common practical application of atomisation is the subsequent combustion of liquid fuels, as very small droplets are mandatory for an efficient combustion. Additionally, stricter environmental policies on pollutants emissions, along with the increasing demand for better energy efficiencies on thermal equipment, created a growing interest on spray flames research. A spray flame is a two-phase flow of liquid fuel droplets and a reacting gaseous mixture composed of vaporised fuel, air and combustion products. The droplet size and velocity distributions play a dominant role on spray flames behaviour, specifically on their efficiency and stability, temperature distribution and pollutants emissions. In particular, as

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showed by Caldas and Semião [4,5], the temperature distribution inside the combustors is particularly influenced by the rate of energy exchanged between the flame and the enclosing walls by radiation. In turn the absorption/emission coefficients and the extinction coefficients of the solid-phase participating media, which determine the rate of radiation heat transferred, are influenced most by the particulate size distribution. In oil flames, that size distribution is determined, among other factors, by the spray fineness attained during the atomisation process at the atomiser exit.

Understanding and ultimately controlling the dynamics of a spray flame requires a comprehensive knowledge of the interaction of the gas flow with individual droplets emerging from the atomiser. The development of a numerical model for spray flames, a possible approach that is becoming very attractive, constitutes one of the most challenging fields of research, as it requires predicting trajectories for evaporating droplets in a turbulent reacting flow, where heat transfer by both radiation and convection and mass transfer processes are extremely important. The ability of such a mathematical model to accurately predict the behaviour of spray flames partly lies in the precision attained when defining the boundary conditions, as concluded by El Banhawy and Whitelaw [6]. Although for some specific conditions, such as highly volatile fuels in turbulent flames, the effect of the droplets velocity may not be determinant, in flames inside combustors burning heavy fuel oils or other fuels exhibiting low volatility, the droplet velocity distribution (hereafter DVD) plays a significant role on the equipment performance. Furthermore, its precise evaluation becomes mandatory for the design of combustion chambers viewing the energy performance optimisation and the reduction of pollutants emissions. Indeed, the characteristic time for a droplet to evaporate in a hotter environment depends on the Reynolds number and on the surrounding gas properties, as pointed out by Chigier [2] and Williams [3]. Since the Reynolds number depends both on the relative velocity between the droplets and the surrounding gas and on the droplet diameter, a correct evaluation for the evaporation time is only achievable by knowing a priori both the DVD and the droplet size distribution (hereafter DSD).

Additionally, even in non-reacting sprays like those used for irrigation in agriculture, the DVD and the DSD play a significant role on the equipment performance. In those cases, a precise evaluation of the distributions is mandatory in order to prevent damaging of the plants located closer to the nozzle and to ensure a more even distribution of water or pesticides.

The above-mentioned widespread use of sprays, together with the need to comply with more efficient and less pollutant systems, has created the need for better understanding the atomisation process and its most influencing physical parameters. Experimental measurement techniques have so far supplied most of the presently available results regarding sprays characterisation (e.g. Refs. [6–8]). These techniques have presently attained quite a mature state and are able to provide fairly accurate results. However, in the last two decades and due to the significantly lower cost and higher versatility exhibited by the mathematical modelling, there has been a vastly increasing interest on the part of atomisation equipment manufacturers in the development of more precise numerical methods capable of accurately predicting new design performances. Existing models applicable to two-phase flows in oil fired combustors make no attempt to predict spray characteristics by initiating calculations of processes within the atomiser gun itself (see, e.g. Refs. [6,7,9–11]). Rather, they rely on the specification of the DSD and of the average velocity for the droplets over a plane near the atomiser exit. An extensive and difficult measurement programme would be required to achieve such specifications, a very demanding task which would be effected for every atomiser and for all operating conditions. Therefore, the development of reliable numerical techniques appears to be the only practicable approach.

Most of the results published so far in the current literature referring to sprays characterisation focus almost entirely on the spray Sauter mean diameter (hereafter SMD), yielding many empirical correlations for its evaluation (e.g. Refs. [12–19]). In the previous works, those of Lefebvre [16,17] and Wang and Lefebvre [18] are referred to pressure jet atomisers, the others being referred to airblast atomisers or to both types of atomisers [12,15].

As for the spray velocity distribution, it has been studied in far less detail, being usually considered that all droplets emerge from the atomiser with a same average velocity. Experimental results exhibit however a totally different evidence: similarly to the droplets diameter, the atomisation of a liquid induces a droplet velocity distribution covering a wide range of velocities, as shown by Presser et al. [20] and Bachalo et al. [21].

Characterisation of sprays at the atomiser exit plane is presently possible through the use of the maximum entropy formalism, as shown by Li and Tankin [22]. This approach avoids the detailed modelling of the atomisation process, providing the results directly at the atomiser exit plane. The droplet size distribution is frequently characterised by the SMD, which expresses the fineness of a spray in terms of its surface area. Similarly, an average velocity, defined as the ratio of the spray kinetic energy to its linear momentum, can be used to characterise the DVD, an approach that is suggested herein.

The present work extends the works of Semião et al. [12] and Li and Tankin [22] addressing the development of a mathematical model based on the use of the maximum entropy formalism and capable of predicting a joint distribution for the size and for the velocity of the spray droplets. Additionally, and using the joint distribution, models to obtain individual distributions for the size and for the velocity of the spray droplets are also derived herein. Correlations for an average velocity for both pressure jet and airblast atomisers based on assumed velocity profiles in the atomiser gun are also presented.

2. Mathematical formulation of a size/velocity joint probability density function

The droplets size and velocity in sprays are crucial parameters required for the fundamental analysis of the transport of mass, momentum and energy in engineering systems. Various distribution functions have been used to fit existing experimental data, the most commonly used ones being the Rosin–Rammler and the Nukiyama–Tanasawa (see, e.g. Ref. [16]). In the present work attention is confined to the latter.

A joint probability density function (hereafter PDF) for the diameter D and for the velocity U, depending on the SMD and on an average velocity of the spray, is determined herein using the concept of information entropy, as introduced by Shannon and Weaver [23].

According to Jaynes [24], the information entropy may be written for a two-variable continuous distribution as:

$$S = -\int\int P(x,y)\log(P(x,y))\,\mathrm{d}x\,\mathrm{d}y. \tag{1}$$

In the present case the entropy of the system is given by Eq. (2), where V is the droplet volume and E stands for the kinetic energy per unit mass of an individual droplet

$$S = -\int\int P(V,E)\log(P(V,E))\,\mathrm{d}V\,\mathrm{d}E. \tag{2}$$

The solution that maximises the entropy of the system must also obey the mathematical and physical constraints established below:

(i) the sum of all probabilities must be unity

$$\int\int P(V,E)\,\mathrm{d}V\,\mathrm{d}E = 1 \tag{3}$$

(ii) the spray mass flow rate must equal the total mass of droplets produced per unit time

$$\int\int P(V,E)\rho_L\dot{n}V\,\mathrm{d}V\,\mathrm{d}E = \dot{M} \tag{4}$$

where $\dot{n}$ is the number of droplets produced per unit time and $\dot{M}$ stands for the liquid mass flux;

(iii) the kinetic energy flux of the liquid at the atomisers exit plane must equal the sum of the kinetic energy of all the droplets produced per unit time

$$\int\int P(V,E)\rho_L\dot{n}E\,\mathrm{d}V\,\mathrm{d}E = \dot{T} \tag{5}$$

where $\dot{T}$ stands for the kinetic energy flux at the atomiser exit.

Performing a variable exchange in Eq. (2)—V (volume) by D (diameter) and E (kinetic energy) by U (velocity)—and using the Lagrange multipliers method to maximise the resultant equation referring to $P(U,D)$, one obtains:

$$P(U,D) = \int\int \frac{\pi^2}{12}D^5U \times\exp\left(-\lambda_0 - \lambda_1 kD^3 - \lambda_2 kD^3\frac{U^2}{2}\right)\mathrm{d}D\,\mathrm{d}U \tag{6}$$

where the parameters λ_0, λ_1 and λ_2 are the Lagrange multipliers that have to be determined.

Assuming that the limiting values for both the droplet size and velocity are zero and infinity, and inserting Eq. (6) into the constraint equations (3)–(5), an equation for the joint PDF is obtained, after the evaluation of the parameters λ_0, λ_1 and λ_2:

$$\frac{\mathrm{d}^2N}{\mathrm{d}D\mathrm{d}U} = \frac{\rho_L^2\dot{n}^2\pi^2UD^5}{12\dot{M}\dot{T}} \times\exp\left[-\frac{\rho_L\dot{n}(\pi/6)D^3}{\dot{M}} - \frac{\rho_L\dot{n}(\pi/6)D^3(U^2/2)}{\dot{T}}\right]. \tag{7}$$

Introducing now the concept of SMD defined in the usual way by (see, e.g. Ref. [22]):

$$\mathrm{SMD} = \frac{\int\int D^3\,\mathrm{d}^2N}{\int\int D^2\,\mathrm{d}^2N} \tag{8}$$

and substituting Eq. (7) into Eq. (8), yields:

$$\mathrm{SMD} = \left(\frac{\dot{M}}{\rho_L\dot{n}(\pi/6)}\right)^{1/3}\Gamma\left(\frac{5}{3}\right)^{-1} \tag{9}$$

where $\Gamma(n)$ is the statistical gamma function.

Defining the average velocity of the spray as the ratio of its kinetic energy to its linear momentum, as expressed by Eq. (10),

$$\bar{U} = \frac{\int\int(U^2/2)\,\mathrm{d}^2N}{\int\int U\,\mathrm{d}^2N} \tag{10}$$

and substituting Eq. (7) into Eq. (10) yields:

$$\bar{U} = \frac{1}{6}\left(\frac{\dot{T}}{\dot{M}}\right)^{1/2} \Gamma\left(\frac{3}{2}\right)^{-2}. \tag{11}$$

Substituting now Eqs. (9) and (11) into Eq. (7), it can be seen that SMD and $\bar{U}$ are the only parameters required to calculate the spray droplets size and velocity joint distribution:

$$\frac{d^2N}{dDdU} = 3\Gamma\left(\frac{3}{2}\right)^{-4}\Gamma\left(\frac{5}{3}\right)^{-6}\frac{U}{\bar{U}}\left(\frac{D}{\text{SMD}}\right)^{5} \times \exp\left[-\Gamma\left(\frac{5}{3}\right)^{-3}\left(\frac{D}{\text{SMD}}\right)^{3} - \frac{1}{2}\Gamma\left(\frac{5}{3}\right)^{-3}\Gamma\left(\frac{3}{2}\right)^{-4} \times \left(\frac{U}{\bar{U}}\right)^{2}\left(\frac{D}{\text{SMD}}\right)^{3}\right]\frac{1}{\text{SMD}}\frac{1}{\bar{U}}. \tag{12}$$

Furthermore, it can be shown that the average velocity $\bar{U}$ is the only parameter necessary to determine the velocity distribution and that the knowledge of the SMD is sufficient to obtain the droplet size distribution (the latter already shown by Semião et al. [12]).

Integrating Eq. (12) over the droplets diameter D, one determines the droplets velocity distribution, given by Eq. (13):

$$\frac{dN}{dU} = \int_0^\infty \frac{d^2N}{dDdU}dD = \Gamma\left(\frac{3}{2}\right)^{-4}\frac{U}{\bar{U}^2}\left[1 + \frac{1}{2}\left(\frac{U}{\bar{U}}\right)^{2}\Gamma\left(\frac{3}{2}\right)^{-4}\right]^{-2}. \tag{13}$$

Proceeding similarly with respect to the droplet size distribution, that is, integrating Eq. (12) over the droplets velocity U, one obtains, as expected, the same result that Semião et al. [12] obtained for the diameter distribution:

$$\frac{dN}{dD} = \int_0^\infty \frac{d^2N}{dDdU}dU = 3\Gamma\left(\frac{5}{3}\right)^{-3}\frac{D^2}{\text{SMD}^3}\exp\left[-\Gamma\left(\frac{5}{3}\right)^{-3}\left(\frac{D}{\text{SMD}}\right)^{3}\right]. \tag{14}$$

In order to solve Eqs. (12)–(14) numerically, the values of the SMD and of the average velocity $\bar{U}$ must be known in advance or explicitly determinable. There is a vast collection of correlations for the SMD available in the literature, for most kinds of atomisers currently used in practical applications over a wide range of operation conditions, as mentioned before (see, e.g. Refs. [12–19]).

Making recourse to those references, in the present work, the SMD for pre-filming airblast atomisers is determined by the semi-empirical correlation of Rizkalla and Lefebvre [13,14] with constants tuned by Jasuja [15]:

$$\text{SMD} = 10^{-3}\left[\frac{\sqrt{\sigma\rho_L}}{\rho_A U_A}\right]\left(1 + \frac{1}{\text{AFR}}\right)^{0.5} + 6\times10^{-5}\left[\frac{\mu_L^2}{\sigma\rho_A}\right]^{0.425}\left(1 + \frac{1}{\text{AFR}}\right)^{0.5} \tag{15}$$

where ρ_L and ρ_A stand for the liquid and air density, respectively, μ_L is the liquid viscosity, σ_L the liquid surface tension, U_A the air velocity and AFR the air/fuel ratio.

Semião et al. [12] presented the following dimensionally consistent correlation for plain jet airblast atomisers that will be used herein, and that was based on the correlation proposed by Jasuja [15] and on the experimental data from Carvalho et al. [25]:

$$\text{SMD} = 1.58\times10^{3}\left[\frac{\sigma}{\rho_A U_A^2 d_0}\right]^{0.5} d_0\left[\frac{\sigma}{\mu_L U_A}\right]^{0.55}\left(\frac{\rho_L}{\rho_A}\right)^{-1} \times\left[1 + \frac{1}{\text{AFR}}\right]^{0.5} + 166\left[\frac{\mu_L}{\rho_L d_0 U_A}\right]^{1.1} \times\left[\frac{\sigma}{\rho_A U_A^2 d_0}\right]^{0.2} d_0\left(\frac{\rho_A}{\rho_L}\right)^{0.35}\left[1 + \frac{1}{\text{AFR}}\right]^{-0.48} \tag{16}$$

where the common variables have the same meanings as in Eq. (15) and d_0 is the discharge orifice diameter.

For pressure-jet atomisation the droplet formation process can be divided into two simplified stages allowing for the derivation of a semiempirical correlation for the SMD. The first phase of the atomisation process represents the generation of surface instabilities, while the second stage is the conversion of those surface protuberances into ligaments and then drops. This simplified approach allowed Lefebvre [17] to postulate the following expression for the SMD that is to be used in the present predictions:

$$\text{SMD} = A\left[\frac{\sigma^{0.5}\mu_L}{\rho_A^{0.5}\Delta p_L}\right]^{0.5}[t\cos(\theta)]^{0.25} + B\left[\frac{\sigma\rho_L}{\rho_A\Delta p_L}\right]^{0.25}[t\cos(\theta)]^{0.75} \tag{17}$$

where σ, μ_L, ρ_L, ρ_A and d_0 have the same meanings as in Eq. (16), Δp_L is the pressure differential across the nozzle, θ the half spray angle and t the film thickness given by:

$$t = 2.7\left[\frac{d_0 \text{FN}\mu_L}{\sqrt{\Delta p_L \rho_L}}\right]^{0.25}. \tag{18}$$

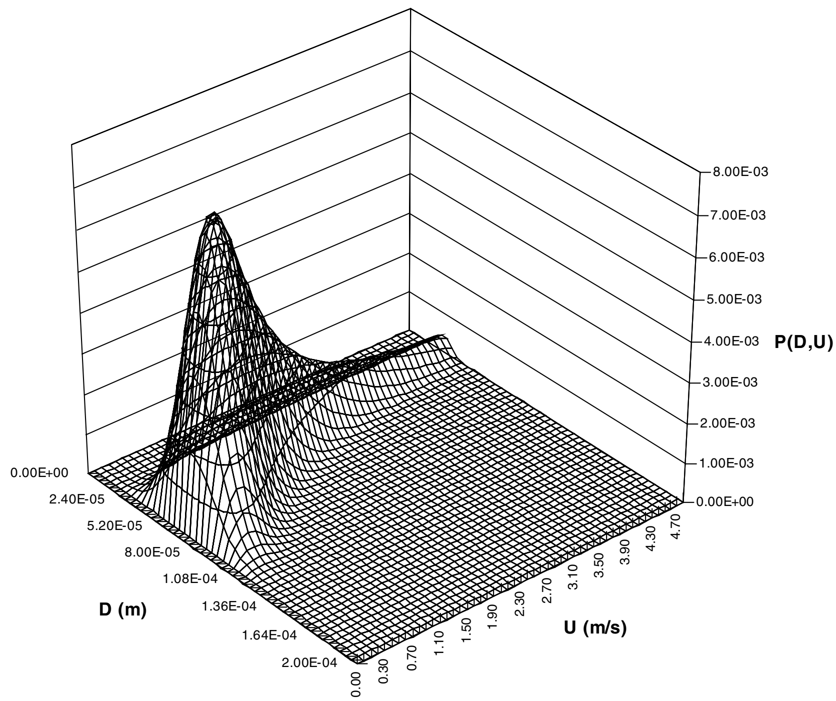


Fig. 1. Droplet size and velocity joint distribution of a spray from a pre-filming airblast atomiser ($U_A = 100$ m s^{-1}, AFR = 1, fuel: kerosene).

In the previous equation FN is the flow number defined as:

$$\mathrm{FN} = \frac{\dot{M}}{\sqrt{\Delta p_L \rho_L}}. \tag{19}$$

The equations to obtain the constants A and B used in this work were adjusted by Semião et al. [12]:

$$A = 2.11[\cos 2(\theta - 30)]^{2.25}\left(\frac{3.4 \times 10^{-4}}{d_0}\right)^{0.4}, \tag{20}$$

$$B = 0.635[\cos 2(\theta - 30)]^{2.25}\left(\frac{3.4 \times 10^{-4}}{d_0}\right)^{0.2}. \tag{21}$$

In opposition to the case of SMD, there is lack of correlations to determine the value of the average velocity $\bar{U}$ depending on easily measurable parameters. For the sake of coherence with the definition expressed by Eq. (10), the evaluation of both kinetic energy and momentum of the liquid flow emerging from the atomiser requires the knowledge of the velocity profiles of the liquid flow inside the atomiser gun.

A possible approach consists of assuming the velocity profile for the flow at the pressure jet atomiser gun as turbulent and obeying to the following variation law (e.g. Ref. [26]):

$$\frac{U}{U_{\max}} = \left(1 - \frac{r}{R}\right)^{1/7} \tag{22}$$

where R is the nozzle radius.

For airblast atomisers the liquid flows at very low Reynolds numbers and, therefore, the flow is assumed to be laminar and exhibits the Hagen–Poiseuille velocity profile:

$$U = -\frac{1}{4\mu_L}\frac{\partial p}{\partial x}(R^2 - r^2) \tag{23}$$

where $\partial p/\partial x$ is the pressure gradient across the atomiser nozzle.

Substitution of Eqs. (22) and (23) into Eq. (10) results in the following equations for the average velocity, expressed in terms of the liquid mass flow rate $\dot{M}$ and of the nozzle diameter d_0, Eq. (24) being valid for pressure jet atomisers whilst Eq. (25) is valid for pre-filming and plain jet airblast atomisers:

$$\bar{U} = \frac{320}{147}\frac{\dot{M}}{\pi \rho_L d_0^2}, \tag{24}$$

$$\bar{U} = \frac{16}{5}\frac{\dot{M}}{\rho_L \pi d_0^2}. \tag{25}$$

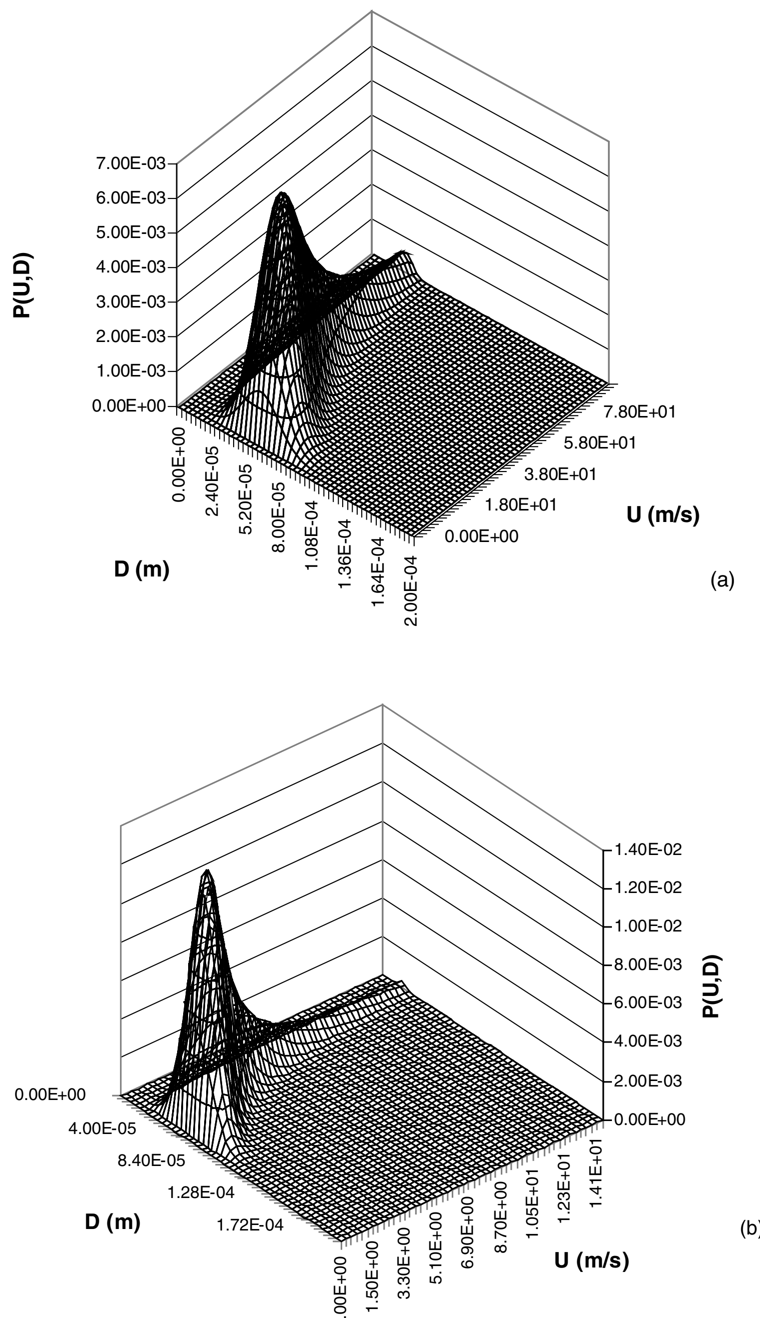


Fig. 2. Droplet size and velocity joint distributions of sprays from: (a) pressure jet atomiser ($FN = 12.5 \times 10^{-8}$, $d_0 = 3.35 \times 10^{-4}$ m, $\Delta P_L = 6.9 \times 10^5$ Pa m^{-1}, $\theta = 30°$, fuel: kerosene); (b) plain jet airblast atomiser ($U_A = 100$ m s^{-1}, $d_0 = 2.5$ mm, AFR = 1, fuel: kerosene).

3. Results

The application of the previously presented correlations for SMD and $\bar{U}$ to practical atomisation devices, together with the use of the predictive equations obtained for the droplet size and velocity distributions, is performed in order to both demonstrate the potential of the developed tool and to validate the results against existing experimental data. The validation is performed for the spray velocity distribution, since the spray size distribution was already validated in a previous work [12]. Additionally, the developed tool is also used for the study of the effect on the spray DSD, DVD and joint size/velocity distribution from changing some process controlling parameters.

Fig. 1 shows the joint distribution for the size and velocity of a spray in a pre-filming airblast atomiser, using a three-dimensional view. This representation allows for the

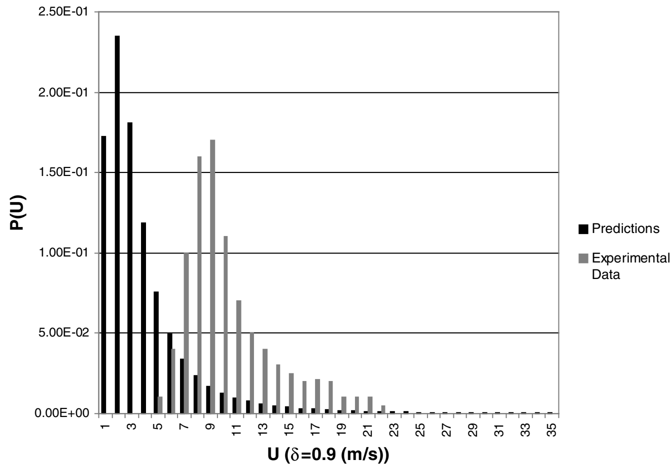


Fig. 3. Comparison of the predicted velocity distribution against experimental data of Presser et al. [20] in a pressure jet atomiser ($\bar{U} = 7.35$ m s^{-1}).

observation of the most striking features of the joint distribution. The relationship and inter-dependence between the size and the velocity of the sprayed droplets are quite evident in Fig. 1. Indeed, in a spray, the values for the size and for the velocity of a droplet that occur are not independent events. Therefore, it may be inferred that the correct evaluation of the percentage of droplets in a spray exhibiting simultaneously a given pair of values for the diameter and the velocity requires the use of the joint PDF, rather than using the value obtained from the product of the probabilities of those parameters to occur individually. This feature, as it can be observed from Fig. 2 that displays a three-dimensional view of the joint distributions for the size and velocity of sprays in a plain jet airblast atomiser and in a pressure jet atomiser, is common to all types of atomisers.

As mentioned earlier, there is a marked lack of available data in the literature for the DVD of atomised sprays, although the works of Presser et al. [20] and Bachalo et al. [21] constitute exceptions. As far as experimental data on joint size/velocity distributions for the droplets in atomised sprays are concerned, and to the authors' knowledge, they are totally non-existent. Therefore, the validation performed herein is somehow limited. Fig. 3 compares the experimental results for the velocity distribution in a pressure jet atomiser obtained by Presser et al. [20] with those obtained using the model derived in this work. The experimental values were measured at a distance of 10 mm downstream the atomiser exit for a swirling combusting flow (with a swirl number of 0.53) and refer to the distribution at the spray axis. The liquid atomised was kerosene. The predictions, in turn, yield results at the atomiser exit, rather than at 10 mm downstream the nozzle, and cover the entire nozzle area, rather than being restricted to the spray axis. It is, therefore, expected that there are occurrences of some quantitative differences between the predicted and the experimental curves for the DVD. These differences can be clearly observed from Fig. 3.

In spite of those quantitative differences, observed between the measurements and the predictions, the main trends for the velocity distribution of the spray are still preserved. Indeed, the sharp asymmetry of the distribution and a pronounced peak for velocity values slightly below the average velocity are present in both measured and predicted distributions.

The referred differences may result from the rotation of the flow. In fact, the swirl imparted to the flow, as mentioned by Presser et al. [20], will promote the tendency for larger droplets to spread away from the spray axis to its edge resulting in a greater density of smaller droplets in the symmetry axis region. Considering that the results obtained by Presser et al. [20] refer to a distribution at the spray axis where smaller droplets prevail in number, the average spray velocity in this region is expected to be higher than the entire average spray velocity (with the larger droplets included). In fact, the average velocity in the smaller diameter range is higher than that occurring in the larger diameter range of a distribution. This fact, besides intuitive, can be observed in all the size/velocity joint distributions depicted in this work (for example, Figs. 1 and 2). From these figures it can also be observed that, regardless of the shifting that occurs in the average velocity, the shape of the DVD remains unchanged. Both features described above are precisely the ones that can be observed in Fig. 3. This corroborates the previous analysis, which yielded the conclusion that the differences between the predicted and the experimental DVD was due to the different regions to which they were referred: the experimental DVD was referred to the spray axis region while the predictions were referred to the entire spray region at the nozzle exit.

It should be mentioned that, besides the swirl effect, another possible cause for the above-mentioned differences

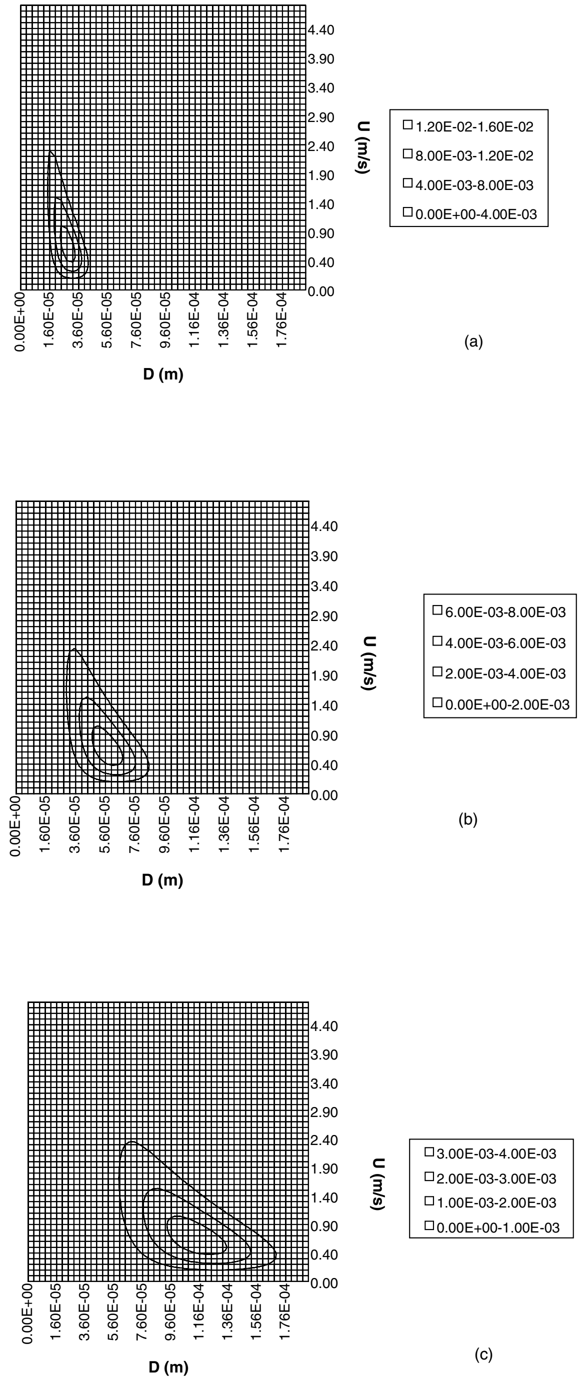


Fig. 4. The effect of the spray fineness—SMD—in a pre-filming airblast atomiser on its velocity distribution ($\bar{U} = 0.9\ \mathrm{m\ s^{-1}}$) : (a) SMD = 30 μm; (b) SMD = 60 μm; (c) SMD = 120 μm.

is the fact that the measurements of Presser et al. [20] were taken for a reacting flow at 10 mm downstream the atomiser nozzle allowing for some evaporation of all the spray droplets to occur. Consequently, the average diameter of the spray at 10 mm downstream the nozzle was probably smaller than that at the nozzle exit.

A rather interesting feature of a spray is the relation existing between the droplets diameters and its velocity values. In order to evaluate this relation, a study of the effect of the spray SMD in the joint distribution was performed. The results are depicted in Fig. 4 for a pre-filming airblast atomiser. As can be observed from this figure the decrease of the spray fineness, i.e. the SMD increase, makes the joint distribution of the spray size and velocity flatter and wider. This means that the spray becomes much less homogeneous as far as both the size and velocity of the droplets are concerned. Indeed, in Fig. 4, as the SMD increases—from (a) to (c)—the distribution moves to the right-hand side of the graphic (larger droplets) and, simultaneously, the range of droplets diameters that occur in the atomisation process becomes wider. Additionally, it is clear from Fig. 4 that after the break-up process the range of droplets diameters for a given velocity increases with the value of the SMD.

Figs. 5 and 6 depict, respectively, the DSD and the DVD for a plain jet airblast atomiser for two different values of the liquid mass flow rate. It can be seen from these figures that an increase of the liquid mass flow generates a spray exhibiting both a lower quality—larger droplets—and an increase of the droplets mean velocity—higher velocities are more likely to occur. Moreover, the velocity distribution presents a sharper asymmetry than that observed for the size distribution. It is clear from Fig. 6 that any droplet in the spray may possess a wider range of velocity values as the mass flow rate increases. The same feature can be observed in Fig. 5 where any droplet may assume a wider range of diameter values as the mass flow rate increases. These tendencies may be confirmed from Fig. 7 that compares the joint probability density functions for the same cases described above.

Comparison between Figs. 4 and 7 reveals an important feature in the atomisation process related to the dependence of the droplets velocities on their sizes. While Fig. 4 displays the results of a parametric study of the joint distribution as a function of the SMD, in which the mean velocity at the atomiser exit was deliberately kept constant, in Fig. 7, the variation of the atomised liquid mass flow rate yielded direct changes in both the values of $\bar{U}$ and the SMD. The above-mentioned dependence is clear in Fig. 4 where the droplets velocities are only affected in an indirect way by the change in their size. However, in the case presented in Fig. 7, the increase in the mass flow rate $\dot{M}$ resulted in a direct increase of both the SMD and $\bar{U}$. It should be noted that this increase in both the SMD and $\bar{U}$ values in a spray produces similar broadening effects in the joint distribution. This constitutes another evidence of the interdependence between the size and velocity of the spray droplets.

Fig. 8 displays the joint distributions for a pressure jet atomiser using water and a residual fuel oil as fluids. The effect of the fluid properties on the distribution can be clearly seen in this figure. The atomisation of the RFO resulted in a lower quality spray, characterised by a larger

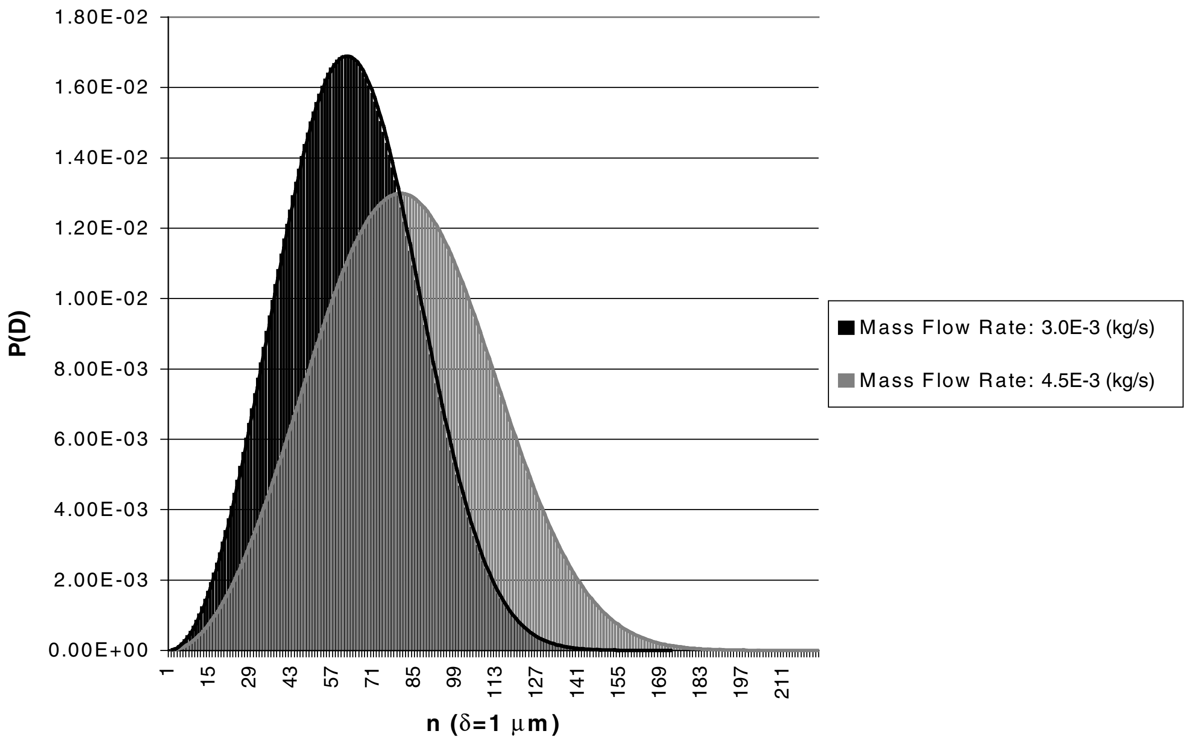


Fig. 5. The effect of the liquid mass flow rate on the droplet size distribution for a plain jet airblast atomiser ($U_A = 100\ \mathrm{m\ s^{-1}}$, $d_0 = 2.5$ mm, AFR = 1, fuel: kerosene).

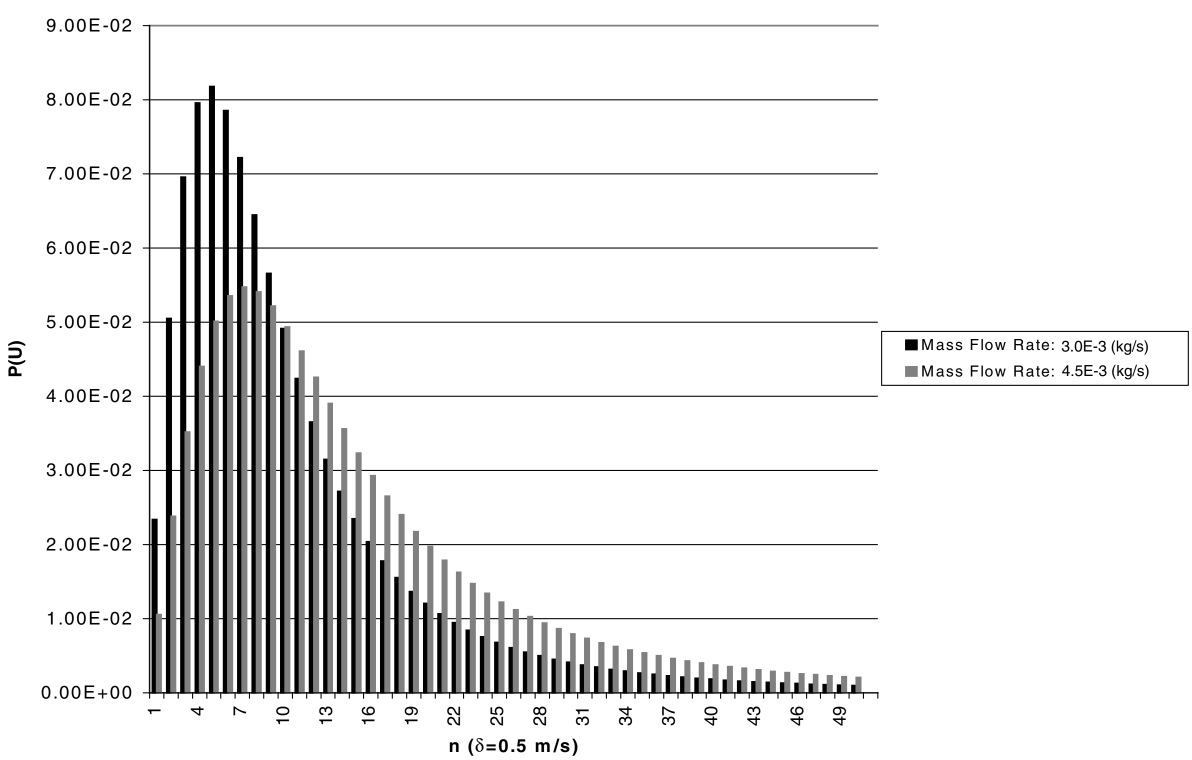
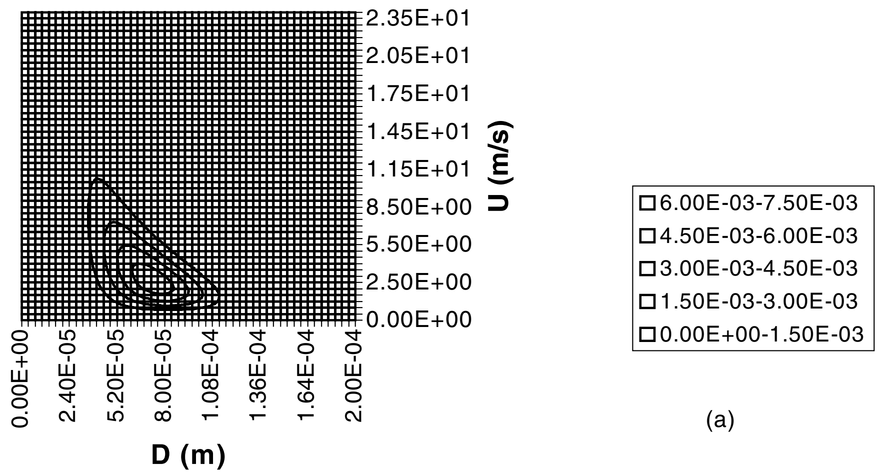


Fig. 6. The effect of the liquid mass flow rate on the droplet velocity distribution for a plain jet airblast atomiser ($U_A = 100\ \mathrm{m\ s^{-1}}$, $d_0 = 2.5$ mm, AFR = 1, fuel: kerosene).



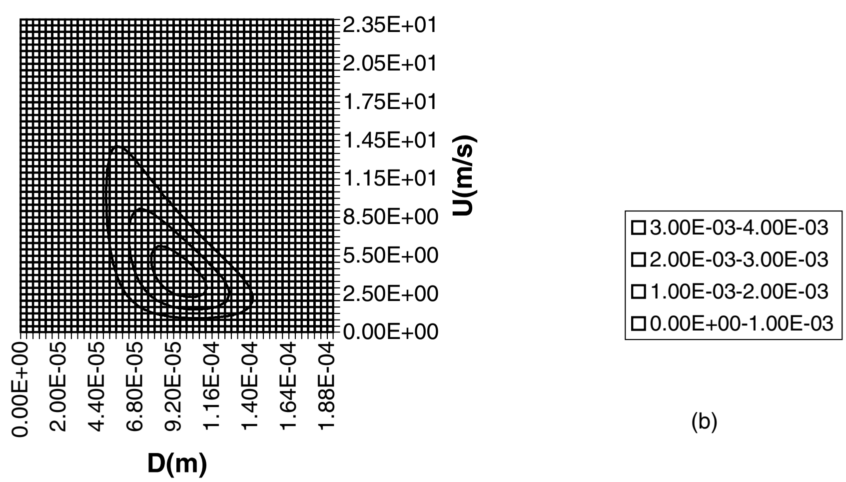


Fig. 7. The effect of the mass flow rate on the droplet size and velocity joint distribution for a plain jet airblast atomiser ($U_A = 100$ m s^{-1}, $d_0 = 2.5$ mm, AFR = 1, fuel: kerosene): (a) $\dot{M} = 3.0 \times 10^{-3}$ kg s^{-1}; (b) $\dot{M} = 4.5 \times 10^{-3}$ kg s^{-1}.

SMD (128 μm), when compared to the one obtained by employing water (with an SMD of 88.5 μm). This is a consequence of the significant impact of the viscosity on the spray quality: larger values of the viscosity yield lower quality sprays as viscosity acts as a counter break-up agent. Although the mean velocity of the spray remains the same (both sprays have $\bar{U} = 20$ m s^{-1}), the changes in the droplets velocities, induced by a variation of the SMD, are significantly visible in this figure.

4. Conclusions

In the present work a mathematical model to predict the joint distribution for the size and velocity of spray droplets based on the maximum entropy formalism was derived. The model was then applied to predict several joint distributions for pressure jet atomisers and for both pre-filming and plain jet airblast atomisers. Using the above-mentioned joint distribution, individual distributions for the size and for the velocity of the spray were also presented. Both joint distribution and individual DSD and DVD are crucial parameters for two-phase flow predictions. The numerical model presented in this work constitutes a powerful tool for the engineering design of atomisers as its use avoids the need for extensive and difficult measurement programmes to obtain the initial conditions for spray flows calculations.

The model results have shown that, when compared

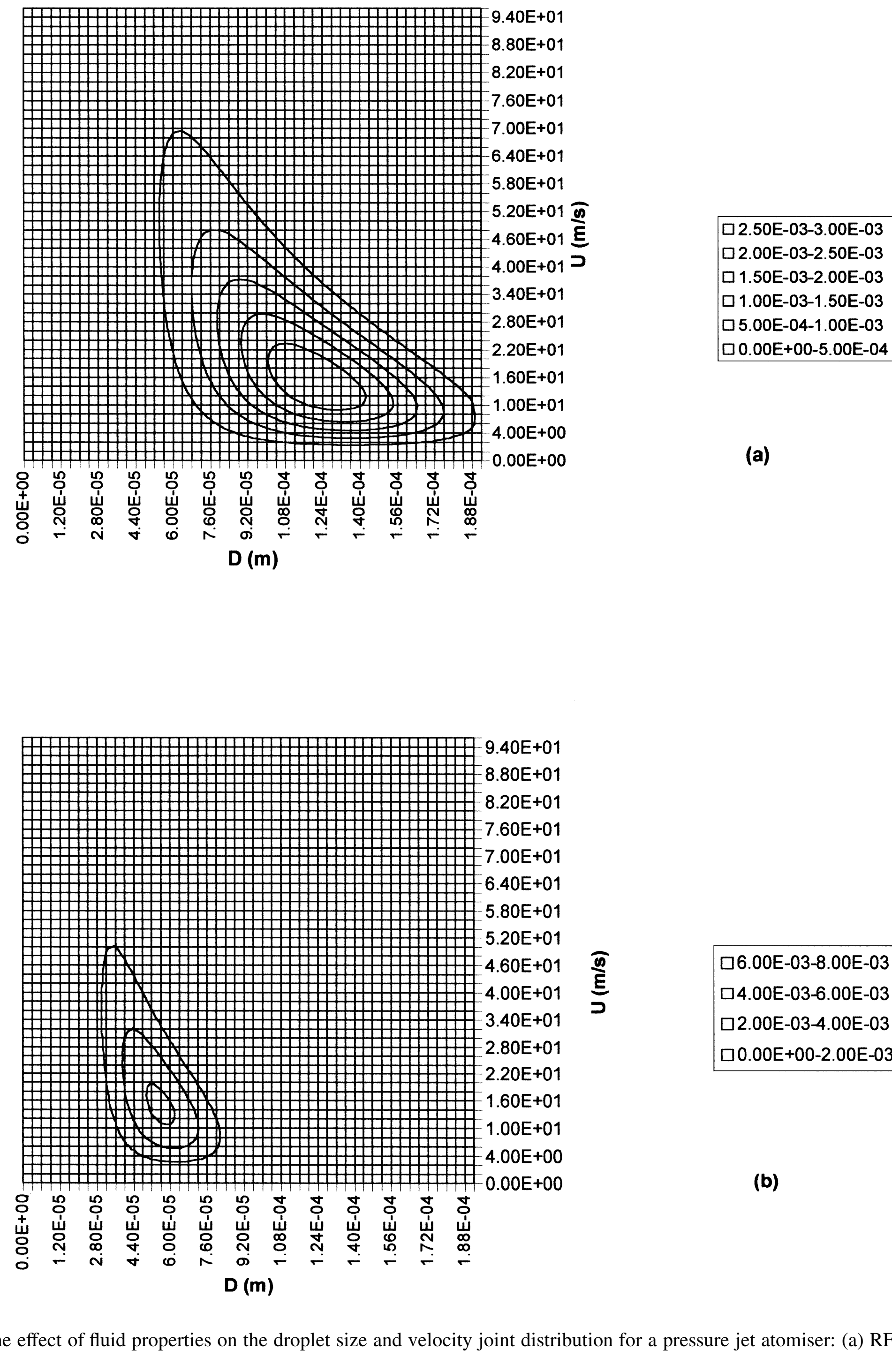


Fig. 8. The effect of fluid properties on the droplet size and velocity joint distribution for a pressure jet atomiser: (a) RFO; (b) water.

to the DSD, the velocity distribution presents both a sharper asymmetry (skewness) and larger peak values (kurtosis), characteristics that were also observed in experimental distributions presented in the current literature.

Additionally, it was shown in this work that, in a spray, the values for the size and for the velocity of a droplet that occurs during the break-up process are not independent events. Therefore, in a spray, the correct evaluation of the probability of droplets to exhibit simultaneously a given pair of values for the diameter and the velocity requires the use of the joint PDF, rather than using the value obtained from the product of the probabilities of those events to occur individually.

Moreover, the predicted distributions indicated that there is a rather interesting feature of sprays, which consists of the loss of homogeneity for both the droplets size and velocity when the spray fineness is decreased (spray with larger droplets). This constitutes a very important result for the engineering design of atomisers.

References

[1] Yuan J, Semião V, Carvalho MG. Int J Energy Res 1997;21:1331.
[2] Chigier NA. Prog Energy Combust Sci 1976;2:97.
[3] Williams A. Prog Energy Combust Sci 1976;2:167.
[4] Caldas M, Semião V. JQSRT 1999;62(4):495.
[5] Caldas M, Semião V. Int J Heat Mass Transfer 1999;42(24):4535.
[6] El Banhawy Y, Whitelaw J. AIAA J 1980;18:1503.
[7] Faeth GM. Prog Energy Combust Sci 1987;13:293.
[8] Shaw SG, Jasuja AK. Atomiz Spray Tech 1987;3:107.
[9] Boysan F, Ayres WH, Swithenbank J, Pan Z. AIAA J Energy 1982;6(6):368.
[10] Gosman AD, Ioannides E. AIAA J Energy 1983;7(6):482.
[11] Wild PN, Boysan F, Swithenbank J. J Inst Energy 1988;61:27.
[12] Semião V, Andrade P, Carvalho MG. Fuel 1996;75:1707.
[13] Rizkalla AA, Lefebvre AH. J Engng Power 1975;97:173.
[14] Rizkalla AA, Lefebvre AH. J Fluids Engng 1975;97:316.
[15] Jasuja AK. J Engng Power 1979;101:250.
[16] Lefebvre AH. Prog Energy Combust Sci 1980;6:233.
[17] Lefebvre AH. Atomiz Spray Tech 1987;3:37.
[18] Wang XF, Lefebvre AH. Atomiz Spray Tech 1987;3:209.
[19] Lorenzetto GE, Lefebvre AH. AIAA J 1987;15:1006.
[20] Presser C, Gupta AK, Semerjian HG. Combust Flame 1993;92:25.
[21] Bachalo WD, Houser MJ, Smith JN. Atomiz Spray Tech 1987;3:53.
[22] Li X, Tankin RS. Combust Sci Tech 1987;56:65.
[23] Shannon CE, Weaver W. The mathematical theory of communication. Urbana, IL: University of Illinois Press, 1949.
[24] Jaynes ET. In: Rosenkrantz RD, editor. Papers on probability, statistics and statistical physics, Dordrecht: Reidel, 1983.
[25] Carvalho MG, Costa MM, Lockwood FC, Semião V. In: Proceedings of the International Conference on Mechanics of Two Phase Flows, 1989. p. 254–60.
[26] White FM. Fluid mechanics. New York: McGraw Hill, 1986.